1. State Rouche’s Theorem.

Look it up in the book.

2. Compute the integral

\[
\int_0^{2\pi} \frac{d\theta}{5 + 4\sin(\theta)}.
\]

Let \( z = e^{i\theta} \). Then \( \sin(\theta) = \frac{z - z^{-1}}{2i} \). Also, \( d\theta = \frac{i}{z} dz \). Hence, the integral (2.1) is equal to

\[
\int_C \frac{1}{5 + 4 \frac{z - z^{-1}}{2i}} \frac{1}{iz} \, dz = \int_C \frac{1}{2z^2 + 5iz - 2} \, dz,
\]

where \( C \) is the unit circle oriented in the positive direction. Since,

\[
2z^2 + 5iz - 2 = 2(z + \frac{i}{2})(z + 2i),
\]

we see that the only singularity of the function under the integral (2.2) within \( C \) is \( z = -\frac{i}{2} \). Therefore, the integral is equal to

\[
2\pi i \text{res}_{z = -\frac{i}{2}},
\]

where

\[
\text{res}_{z = -\frac{i}{2}} = \frac{1}{2(z + 2i)} \bigg|_{z = -\frac{i}{2}} = \frac{1}{3i}.
\]

Therefore the integral (2.1) is equal to \( \frac{2\pi}{3} \).


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