ODE sec. 5, Spring 2008

Homework 9

1. Find the radius of convergence of the series

$$\sum_{n=0}^{\infty} \frac{1}{2^n} x^n.$$

Let R be the radius of convergence of the series. Then

$$\frac{1}{R} = \lim_{n\to\infty} \left(\frac{1}{2^{n+1}}\right) / \left(\frac{1}{2^n}\right) = \frac{1}{2}.$$

Thus $R = 2$.

2. Find the radius of convergence of the series

$$\sum_{n=0}^{\infty} n^n x^n.$$

Let R be the radius of convergence of the series. Then

$$\frac{1}{R} = \lim_{n\to\infty} (n^n)^{\frac{1}{n}} = \lim_{n\to\infty} n = \infty.$$

Thus $R = 0$. Hence the series converges for $x = 0$ only.

3. Express the series as a combination of elementary functions, such as polynomials, e^x, $sin(x)$ or $cos(x)$:

$$\sum_{n=2}^{\infty} \frac{1}{n!} x^n.$$

$$\sum_{n=2}^{\infty} \frac{1}{n!} x^n = \sum_{n=0}^{\infty} \frac{1}{n!} x^n - \frac{1}{0!} x^0 - \frac{1}{1!} x^1 = \sum_{n=0}^{\infty} \frac{1}{n!} x^n - 1 - x = e^x - 1 - x.$$

4. Express the series as a combination of elementary functions, such as polynomials, e^x, $sin(x)$ or $cos(x)$:

$$\sum_{n=3}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}.$$

$$\sum_{n=3}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}$$
$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1} - \frac{(-1)^0}{(2\,0+1)!} x^{2\,0+1} - \frac{(-1)^1}{(2\,1+1)!} x^{2\,1+1} - \frac{(-1)^2}{(2\,2+1)!} x^{2\,2+1}$$
$$= sin(x) - x + \frac{1}{3} x^3 - \frac{1}{5} x^5.$$

5. Find the general solution of the equation

$$(x^2 - 4)y'' = -3xy' - y.$$

See example 5 in section 8.2, page 512 in the book (3rd edition).