ODE sec. 5, Spring 2008

Homework 10, solutions

1. Use Laplace transform to solve the following initial value problem:

$$y''(x) + 4y(x) = 4x, \quad y(0) = 1, \quad y'(0) = 5.$$

By taking the transform of both sides we get the following equation:

$$(s^2 + 4)Y(s) = s + 5 + \frac{4}{s^2}.$$

Hence,

$$\begin{aligned} Y(s) =& \frac{s}{s^2+4} + \frac{5}{s^2+4} + \frac{4}{s^2(s^2+4)} \\ =& \frac{s}{s^2+4} + \frac{5}{s^2+4} + \frac{1}{s^2} - \frac{1}{s^2+4} \\ =& \frac{s}{s^2+4} + \frac{4}{s^2+4} + \frac{1}{s^2}. \end{aligned}$$

Therefore,

$$y(x) = cos(2x) + 2\,sin(2x) + x.$$

2. Use Laplace transform to solve the following initial value problem:

$$y''(x) + 2y'(x) + 5y(x) = 3e^{-x}sin(x), \quad y(0) = 0, \quad y'(0) = 3.$$

By taking the transform of both sides we get the following equation:

$$(s^2 + 2s + 5)Y(s) = 3 + \frac{3}{(s+1)^2+1}.$$

Hence,

$$\begin{aligned} Y(s) =& \frac{3}{s^2+2s+5} + \frac{3}{(s^2+2s+2)(s^2+2s+5)} \\ =& \frac{3}{s^2+2s+5} + \frac{1}{s^2+2s+2} - \frac{1}{s^2+2s+5} \\ =& \frac{3}{(s+1)^2+4} + \frac{1}{(s+1)^2+1}. \end{aligned}$$

Therefore,

$$y(x) = e^{-x}\,sin(2x) + e^{-x}\,sin(x).$$